Statistics

Lecture 8

Feb 19-8:47 AM

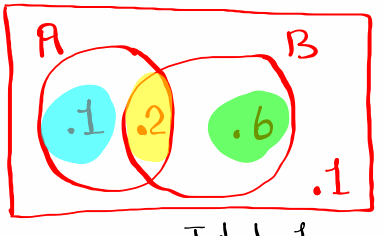
Class QZ 9 Box Your Final Ans.

Given $P(A) = .3$ $P(B) = .8$ $P(A \text{ and } B) = .2$

1) Construct Venn Diagram

$.3 - .2 = .1$

$.8 - .2 = .6$



Total = 1

2) $P(\bar{A}) = 1 - P(A)$

$= 1 - .3 = .7$ ✓

3) $P(A \text{ or } B)$

$= P(A) + P(B) - P(A \text{ and } B)$

$= .3 + .8 - .2 = .9$ ✓

4) $P(A \text{ only or } B \text{ only}) = .1 + .6$

$= .7$ ✓

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Ch. 1

Data
- 1) Qualitative (Non-Numerical)
- 2) Quantitative (Numerical)
  - 1) Discrete (Countable)
  - 2) Continuous (Measureable)

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Let x be a discrete random variable with Prob. dist. $P(x)$.

What is Prob. dist.?

It is a method to give the prob. of all possible outcomes.

It could be in the form of a table, chart, graph, or formula

Some rules:

1) $0 \le P(x) \le 1$
2) $\sum P(x) = 1$
3) $P(x) = 1 \iff$ Sure event
4) $P(x) = 0 \iff$ Impossible event
5) $0 < P(x) \le .05 \iff$ Rare event

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Consider the chart below:

x	$P(x)$
1	.2
2	.5
3	.3

1) Verify $\sum P(x) = 1$

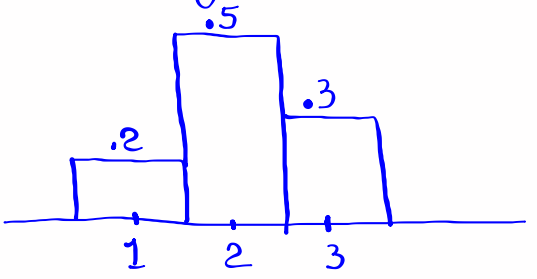
$.2 + .5 + .3 = 1$ ✓

2) $P(x \geq 2) = .5 + .3 = .8$

3) Draw Prob. dist. histogram.

$x \rightarrow$ Midpoint

$P(x) \rightarrow$ Rel. F.



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A box has 2 Dimes and 3 Nickels.

Take 2 Coins with replacement

DD

DN

ND

NN

Let x be # of dimes

DD $\rightarrow x = 2$ $\quad P(x=2) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25} = .16$

DN
ND $\rightarrow x = 1$ $\quad P(x=1) = 2\left(\frac{2}{5} \cdot \frac{3}{5}\right) = \frac{12}{25} = .48$

NN $\rightarrow x = 0$ $\quad P(x=0) = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25} = .36$

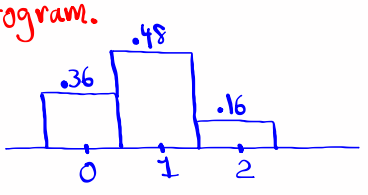
x	$P(x)$
0	.36
1	.48
2	.16

1) Verify $\sum P(x) = 1$

$.36 + .48 + .16 = 1$

2) $P(x \leq 1) = .48 + .36 = .84$

3) Draw Prob. dist. histogram.



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4 Women, 6 Men, Select 2 different people.

WW
WM
MW
MM
} Sample Space

Let x be # of women

$x=0$ $P(x=0)=P(MM)=\frac{6}{10}\cdot\frac{5}{9}=\frac{30}{90}$

$x=1$ $P(x=1)=P(WM \text{ or } MW)=\frac{48}{90}$

$x=2$ $P(x=2)=P(WW)=\frac{4}{10}\cdot\frac{3}{9}=\frac{12}{90}$

x	$P(x)$
0	$\frac{30}{90}$
1	$\frac{48}{90}$
2	$\frac{12}{90}$

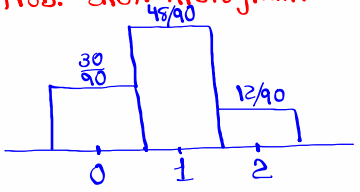
P(both people are Same gender)

$P(MM \text{ or } WW)=\frac{30}{90}+\frac{12}{90}=\frac{42}{90}=\boxed{\frac{7}{15}}$

$P(\text{at least 1 Woman})=1-P(\text{No Woman})$

$=1-P(MM)=1-\frac{30}{90}=\boxed{\frac{2}{3}}$

Prob. dist. histogram:



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Consider the chart below

x	$P(x)$
1	.1
2	.3
3	.4
4	.2

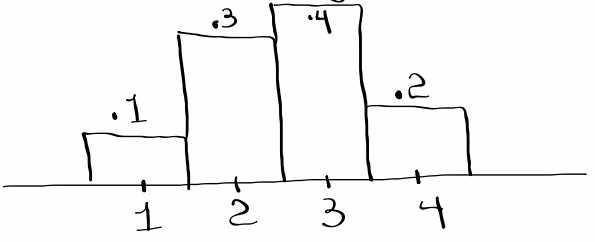
1) $P(x=4)$

$=1-[.1+.3+.4]=1-.8=\boxed{.2}$

2) $P(x=2 \text{ or } x=4)$

$=.3+.2=\boxed{.5}$

3) Draw Prob. dist. histogram.



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Complete the chart below:

x	$P(x)$	$xP(x)$	$x^2P(x)$
1	.3	.3	.3
2	.5	1.0	2.0
3	.2	.6	1.8

1) $\sum xP(x) = 1.9$

2) $\sum x^2P(x) = 4.1$

3) Compute $\sum x^2P(x) - (\sum xP(x))^2 = 4.1 - 1.9^2 = .49$

4) $\sqrt{\text{Lastanswer}} = \sqrt{.49} = .7$

Clear all lists

$x \to L1$, $P(x) \to L2$

L1	L2
1	.3
2	.5
3	.2

$\sum x = 1.9$

$\sum x^2 = 4.1$

$n = 1$ ← Total Prob.

STAT → CALC

1:1-Var Stats

with Menu
List: L1
FreqList: L2
Calculate

No Menu
L1, L2
Enter

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Consider the chart below

x	$P(x)$
2	.1
4	.3
6	.4
8	.2

$x \to L1$, $P(x) \to L2$

STAT → CALC

1:1-Var stats

Use L1 & L2

$\sum x = 5.4$

$\sum x^2 = 32.4$

$n = 1$

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Mean μ "mu"

Variance σ^2 "sigma 2"

Standard deviation σ "sigma"

$$\mu = \sum x p(x) \qquad \sigma^2 = \sum x^2 p(x) - \mu^2 \qquad \sigma = \sqrt{\sigma^2}$$

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Consider the chart below

x	$P(x)$
1	.05
2	.15
3	.25
4	.35
5	.20

$x \rightarrow L1$, $P(x) \rightarrow L2$

Use 1-Var Stats with L1 & L2.

$\mu = \bar{x} = 3.5$

$\sigma = \sigma_x = 1.118$

$n = 1$

what about σ^2?

VARS | 5: Statistics | 4: σ_x | x^2 | Math | 1: ▶Frac | Enter

$$\sigma^2 = \frac{5}{4}$$

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4 W , 6 M , Select 3 people

Let x be # of Women

W W W

Some W & Some M

M M M

$P(x=0) = \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{4}{8} = \frac{120}{720}$

$P(x=1) = 3\left(\frac{4}{10} \cdot \frac{6}{9} \cdot \frac{5}{8}\right) = \frac{360}{720}$

W M M

M W M

M M W

$P(x=2) = 3\left(\frac{4}{10} \cdot \frac{3}{9} \cdot \frac{6}{8}\right) = \frac{216}{720}$

W W M

W M W

M W W

$P(x=3) = \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8} = \frac{24}{720}$

x	P(x)
0	120/720
1	360/720
2	216/720
3	24/720

$x \rightarrow L1$, $P(x) \rightarrow L2$

Use 1-Var Stats with L1 & L2

$\mu = \bar{x} = 1.2$

$\sigma = \sigma_x = .748$

$n = 1$

Find σ^2 in reduced fraction.

$\sigma^2 = \frac{14}{25}$

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Expected Value $\rightarrow \mu \rightarrow \bar{x}$

25 TKTS Sold for $10 each

One TKT drawn $\rightarrow$ winner gets a Calc. worth $100.

Net Pay	P(Net Pay)
10 - 100	1/25
10 - 0	24/25

Win TKT Net pay $\rightarrow$ L1

Losing TKT P(Net pay) $\rightarrow$ L2

Use 1-Var Stats with L1 & L2

House makes $6 Per TKT.

$\mu = \bar{x} =$ $6 E.V. Per TKT

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Pay me $5

Draw one card from a standard deck of playing cards.

If you draw, I give you

Ace	$20
Face	$5
otherwise	Nothing

Net Pay	P(Net Pay)	
5 − 20	4/52	Ace
5 − 5	12/52	face
5 − 0	36/52	Any other card

$E.V. = \mu = \bar{x} \approx \$2.31$

SG 14 & 15 ✓

Net Pay → L1

P(Net Pay) → L2

1-Var Stats L1, L2

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Binomial Prob. dist.: SG 16

1) There are n independent events.

2) Each event has only two outcomes.

$P(\text{Success}) = p$ $P(\text{Failure}) = q$

$p + q = 1$

p & q remain unchanged for all events.

3) $x \rightarrow$ # of successes

$n - x \rightarrow$ # of failures

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

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Consider a binomial Prob. dist with 12 events and $p = .6$

1) $n = 12$ 2) $p = .6$ 3) $q = 1 - p = .4$

4) np
$= 12(.6)$
$= 7.2$

5) npq
$= 12(.6)(.4)$
$= 2.88$

6) $\sqrt{npq}$
$= \sqrt{2.88}$
$= 1.697$

7) $_{12}C_5$ 12 MATH PRB 5 Enter 792
3: $_nC_r$

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Consider a binomial Prob. dist. with 25 trials and .8 prob. of success per trial.

1) $n = 25$ 2) $p = .8$ 3) $q = 1 - p = .2$

4) np
$= 25(.8) = 20$

5) $npq = 25(.8)(.2)$
$= 4$

6) $\sqrt{npq}$
$= \sqrt{4} = 2$

7) $_{25}C_{18}$
25 MATH PRB 3: $_nC_r$ 18 Enter 480 700

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Suppose we flip a fair coin 10 times.

Success is to land tails.

1) $n = 10$ 2) $p = .5$ 3) $q = .5$

P(land exactly 6 tails)

$= P(x = 6) = {}_{10}C_{6} \cdot (.5)^{6} \cdot (.5)^{4} = .205$

(x over the 6, $n - x$ over the 4; n, x, p, q under the 10, 6, .5, .5)

Using TI Command

2nd VARS ↓↓···↓ ▶binompdf(

with Menu
Trials: 10
P: .5
x value: 6
Paste Enter

No Menu
10, .5, 6 Enter

$P(x=6) = \text{binompdf}(10, .5, 6)$
$= .205$

Your work

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You are taking a quiz with 20 questions. Each question has 4 choices with only one correct choice.

You are making random guesses.

Success is to guess correct Answer.

1) $n = 20$ 2) $p = \frac{1}{4} = .25$ 3) $q = \frac{3}{4} = .75$

P(guess exactly 8 correct answers)

$P(x=8) = {}_{20}C_{8} \cdot (.25)^{8} \cdot (.75)^{12} = .061$

(x over the 8, $n - x$ over the 12; n, x, p, q under the 20, 8, .25, .75)

$P(x = 8) = \text{binompdf}(20, .25, 8) = .061$

2nd VARS ↓↓

P(guess at most 8 correct answers)

$P(x \leq 8) = \text{binomcdf}(20, .25, 8) = .959$

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Consider a binomial Prob. dist. with $n=100$ & $P=.5$.

1) $q = 1-P$
$= .5$

2) $np = 100(.5)$
$= 50$

3) $npq = 100(.5)(.5)$
$= 25$

4) $\sqrt{npq} = \sqrt{25} = 5$

P(exactly 55 Successes) $= P(x=55)$
$=$ binompdf(100, .5, 55)
$=$ .048

P(fewer than 55 Successes) $= P(x<55)$
at most 54
$= P(x \leq 54)$
$=$ binomcdf(100, .5, 54)
$=$ .816

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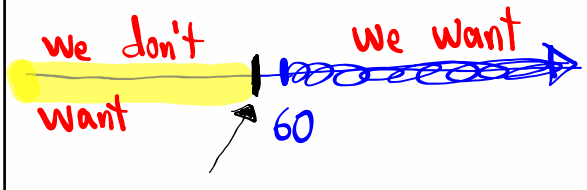
P(at least 60 Successes) $= P(x \geq 60)$



$= 1 - P(x \leq 59)$
↑ Total Prob.
$= 1 -$ binomcdf(100, .5, 59)
$=$ .028

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250 voters were randomly selected.

Suppose prob. that any voter is in support of abortion is .6.

1) $n = 250$ 2) $p = .6$ 3) $q = .4$

4) $np = 150$ 5) $npq = 60$ 6) $\sqrt{npq} \approx 7.746$

P(exactly 180 of them support abortion)

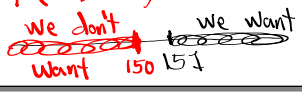
$P(x = 180) = \text{binompdf}(250, .6, 180) = \boxed{2.2 \times 10^{-5}}$

P(at most 165 of them support abortion)

$P(x \leq 165) = \text{binomcdf}(250, .6, 165) = \boxed{.978}$

P(more than 150 of them support abortion)

$P(x > 150) = P(x \geq 151) = 1 - P(x \leq 150)$



$= 1 - \text{binomcdf}(250, .6, 150)$

$= \boxed{.476}$

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